The current state of COVID-19 in Colorado and projected course of the epidemic in the coming weeks

5/29/2020

Prepared by the Colorado COVID-19 Modeling Group
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Summary / Key findings

- Mobility data show a continued decline in time spent at home, likely reflecting the change from Safe at Home to Safer at Home.
- The updated model findings, which cover the period through 5/14, indicate that COVID-19 continues to decline in Colorado, but the declines have begun to slow. The reproductive number remains below 1, indicating the number of new cases per day is declining. However, the extent of social distancing dropped to 75% across 5/9 through 5/14 as the entire state moved to Safer at Home and the reproductive number has increased.
- We estimate approximately 170,000 Coloradoans have been infected with SARS-CoV-2, approximately 2.9% of the population. The proportion of symptomatic infections detected and reported to state surveillance systems appears to be increasing.
- Modeling projections that extend into the summer and through November 2020 continue to indicate a need for social distancing of at least 65% unless most people 60 years and older maintain high levels of social distancing as seen during the Stay at Home period (80%).
- Relaxation of social distancing to lower levels (45%) is predicted to lead to a surge in sick people in excess of hospital capacity by mid-summer, even if implemented with mask wearing, increased case detection and isolation, and higher levels of social distancing by all older adults.
- While the model does not yet incorporate the effect of contact tracing, we did estimate the number of symptomatic incident cases potentially needing contact tracing is approximately 500 at present and that number will likely increase in the coming months as social distancing is relaxed.
- Model findings indicate that increased mask wearing would be beneficial for controlling the COVID-19 epidemic. We lack needed data on the extent of mask use by Coloradans.

Introduction

This is an update to a report published on 5/23/2020 [1]. The goals of this report are to provide

1. Updated estimates on the current state of COVID-19 in Colorado and the cumulative impact of interventions to date.
2. Updated estimates of the number of people infected to date and the proportion of symptomatic cases being detected
3. Projections of the future course of COVID-19 under scenarios including relaxation of social distancing and increases in mask wearing, and the implications for contact tracing.

Part 1. Current state of COVID-19 in Colorado estimated cumulative impact of interventions to date

We used our age-structured SEIR model and hospital census data reported through 5/27/2020 to characterize the current status of the COVID-19 epidemic in Colorado and the collective impact of efforts to reduce the spread of the SARS-CoV-2 virus. This is an update to the report published on 5/23/2020 [1].

The model presently includes three interventions that have been implemented in Colorado to slow the spread of infections: 1) social distancing; 2) mask wearing; and 3) case isolation. We anticipate that future models will include contact tracing, which is being implemented and increased. We use our model output to estimate the effective reproductive number – the average number of infections directly generated by each case – which provides an approximation of the cumulative impact of these interventions to date. Because there is an approximately 13-day lag, on average, between infection and hospitalization, the estimates of the effective reproductive number and of the degree of social distancing relate to the state of COVID-19 transmission through May 14 (this interval includes an average 5-day incubation period and average 8 days between the onset of infectiousness and hospitalization). Information on parameter estimates and methods are provided in the Methods Appendix, below.

Interventions included in the model of COVID-19 to date.

Case isolation. We assume that 53% of symptomatic cases self-isolate 48 hours after the onset of infectiousness and this isolation primarily reduces the spread of infection to members outside of an individual's household. Case isolation is assumed to begin on March 5, the date the first COVID-19 cases were reported in Colorado, and that percentage may include people who have symptoms but have not tested positive. Our estimate of the proportion of cases that self-isolate is based on prior model fits (Methods Appendix Table A1).

Social distancing. Social distancing measures were adopted in Colorado starting in mid-March, to reduce person-to-person contacts and slow the spread of infection. We model social distancing, using a parameter that describes the percent decrease in effective contacts between susceptible and infectious individuals. This parameter accounts for social distancing policies intended to avoid contact altogether (e.g., through workplace and school closures) as well as policies and individual behaviors to reduce potential contact with the virus (e.g., maintaining at least 6 feet of distance between people outside of one's household, and handwashing). To account for the changes in the degree of social distancing over time in relation to policy measures, we now model four phases of social distancing.

- Phase 1 (3/17-3/25): Social distancing measures were implemented in Colorado starting in mid-March in Colorado. Ski resorts were closed on 3/14, many schools by 3/16, and bars, restaurants, theatres and casinos were closed 3/17. For the purpose of the model, we assign a start date of 3/17. We use prior model fits estimating that the phase 1 social distancing parameter was approximately 52%.
- Phase 2 (3/26-4/26): State-wide stay at home order, which began on 3/26 and ended on 4/26. We use prior model fits based on hospitalization data through 5/11/2020 – the best parameter value fit to the data suggests the phase 2 social distancing parameter is approximately 80%.
- Phase 3 (4/27-5/8): Safer at Home for half of Colorado. Colorado transitioned to Safer at Home on 4/27 but six of the seven counties of the Denver metro region, comprising approximately 50% of the Colorado population were under a regional stay at home policy through 5/8. We estimated the best fitting parameter based on the current hospitalization data. **The best fit social distancing parameter for this phase is currently estimated to be 85%.**
- Phase 4 (5/9-5/14): Safer at Home state-wide. As of 5/9, all of Colorado was under Safer at Home policies, while some counties have been granted variances that allow for opening of additional businesses across this period. Due to the aforementioned lags between infection and hospitalization, we are currently only able to estimate the level of social distancing during full Safer-at-Home through 5/14 (using data through 5/27). **The best fit social distancing parameter for this phase is currently estimated to be 75%, indicating a decrease in social distancing.**

The level of reduction in contact rate to date is assumed to be equal across all age groups.

Mask wearing. As of April 4, Coloradoans had been advised to wear masks in public spaces. Masks are intended to trap droplets containing viral particles shed by infectious individuals. We model the effectiveness of mask wearing as a reduction in transmission by asymptomatic and pre-symptomatic individuals (details in the Methods Appendix, below). We use prior model fits estimating approximately 40% of the population has been wearing masks in public beginning on 4/4, coinciding with the Governor's press conference advising Coloradoans to wear masks. We assumed a further increase in mask wearing to 50% of the population beginning on 4/27, corresponding with orders to wear masks in businesses in most major metropolitan areas in Colorado.

Changes in population movement in relationship to social distancing orders

We used anonymized and aggregated mobile device data to evaluate the relationship between social distancing interventions and the amount of time people spend at home (for further details see [2]). These data are available aggregated at the Census block group level.

Figure 1 shows that time spent at home, an indirect indicator of mobility and contact with others, began increasing from the start of the epidemic in early March and continued to increase following the Phase 1 closures and the Phase 2 stay at home order. The amount of time spent at home began to decline in mid-April, ahead of the expiration of the Stay at Home order on April 26 and has continued to decline through the month of May. We caution that a decline of time at home does not necessarily indicate that more time is spent in ways that increase transmission risk.

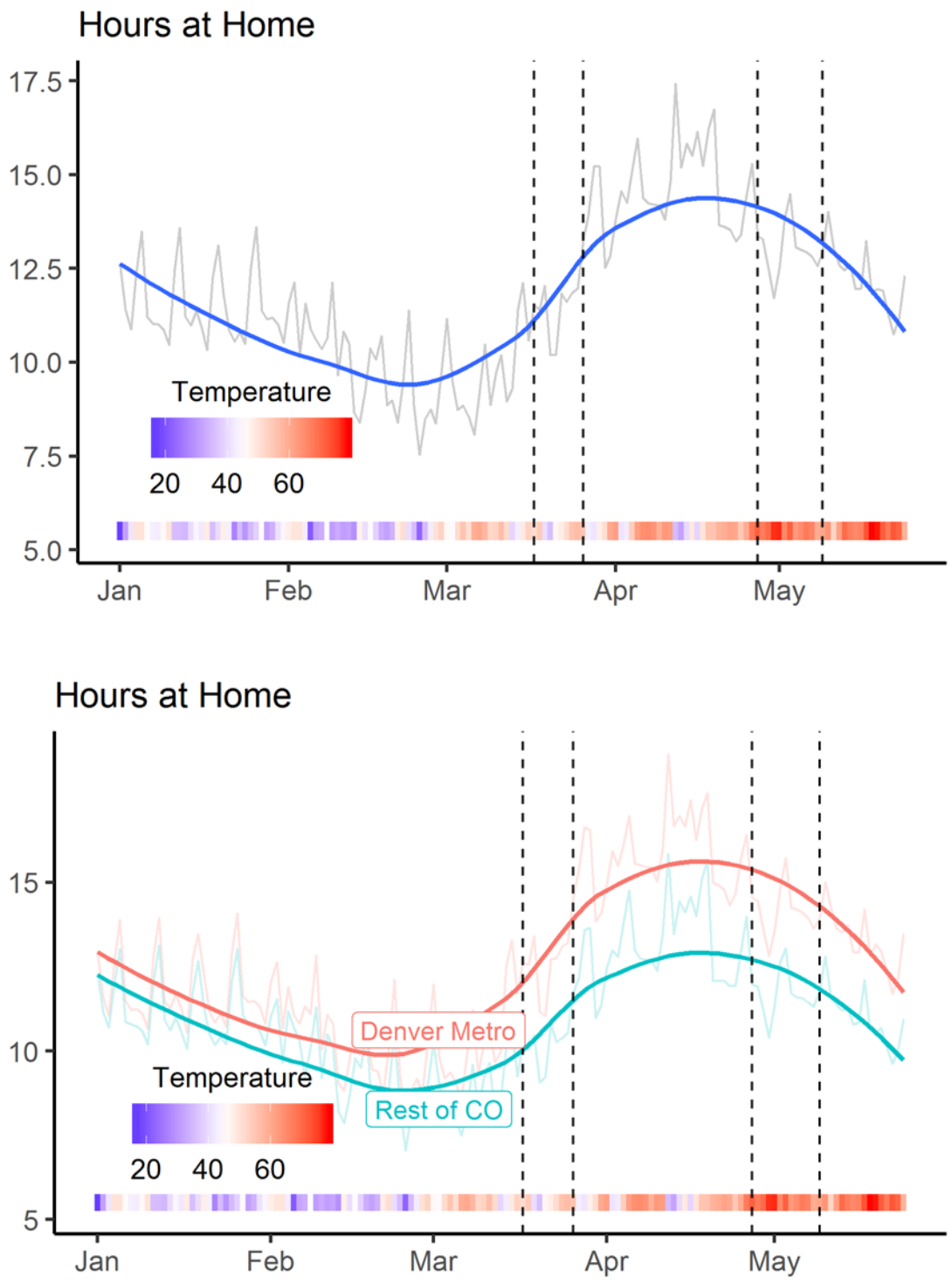

Figure 1. The median number of hours spent at home based on SafeGraph from 1/01/20 to 5/24/20 for all of Colorado (top panel) and stratified by the Denver metro region (bottom panel). Dashed lines indicate the beginning of social distancing measures in mid-March (assigned a date of 3/17), the start of the state-wide stay at home order (3/26), the state-wide transition to Safer at Home (4/27) and the end of the Denver metro region Stay

at Home order and transition to Safer at Home (5/9). The color ribbon indicates CO average maximum daily temperature where blue is cool and red is warm (white is mean 45 degrees F).

Estimated cumulative impact of interventions to date

The number of hospitalized COVID-19 patients in Colorado plateaued in early April and has declined since mid-April (Figure 2). Our updated data shows that the combination of mask wearing, case isolation and high levels of social distancing has led to declines in COVID-19 through the state-wide end of Stay at Home in on 4/26 and in the Denver metro region on 5/8. We estimate the overall impact of the interventions to slow the spread of COVID-19 to date has been to reduce the reproductive number below 1 from late March through 5/14 (Figure 3). This decline in the reproductive number underlies the drop in the number of cases. We cannot yet estimate the reproductive number beyond 5/14.

Our early estimates of the transition to Safer at Home by all of Colorado on 5/9 indicates a modest decline in social distancing through 5/14 to approximately 75%. Given the lags described above, we are only able to examine approximately five days of data after the end of the extended stay at home order for the Denver metro region so that these estimates are uncertain. However, the decline in social distancing would be anticipated as policies change and it is consistent with the progressive decrease in time spent at home. Moreover, while we model social distancing shifts based on the broad policy changes, we note that in reality, Colorado has gradually re-opened businesses over the month of May. For example, retail opened on 5/4 and restaurants on 5/27.

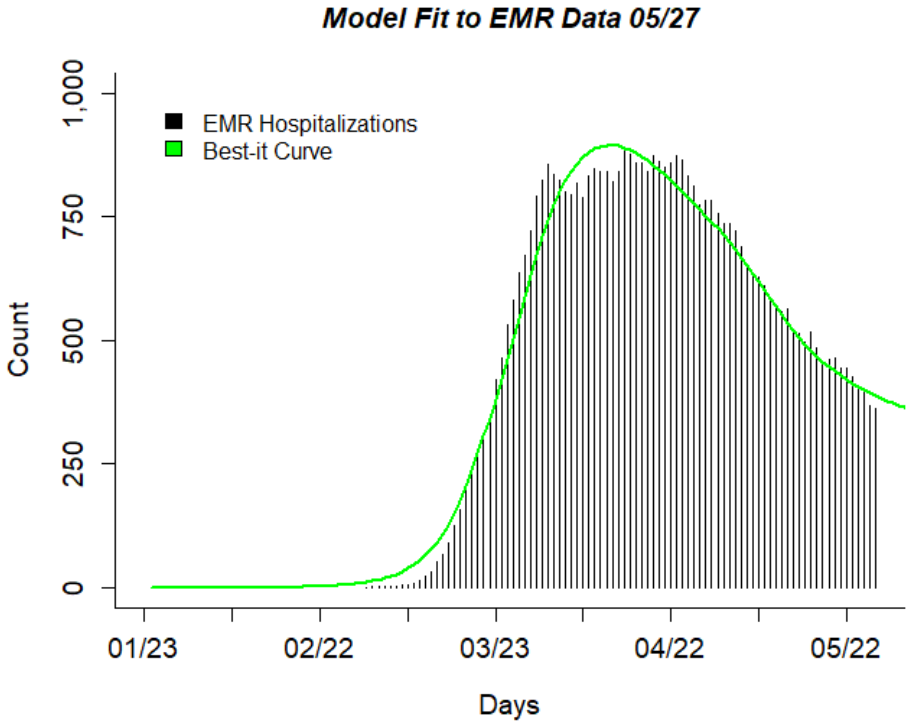

Figure 2. Current model fit (green line) to hospitalized COVID-19 cases (black lines) using the age-structured SEIR model. Hospitalized COVID-19 cases are from CDPHE reported COVID-19 hospitalizations and EMResource (EMR) hospital census data provided by CDPHE through 5/27/2020.

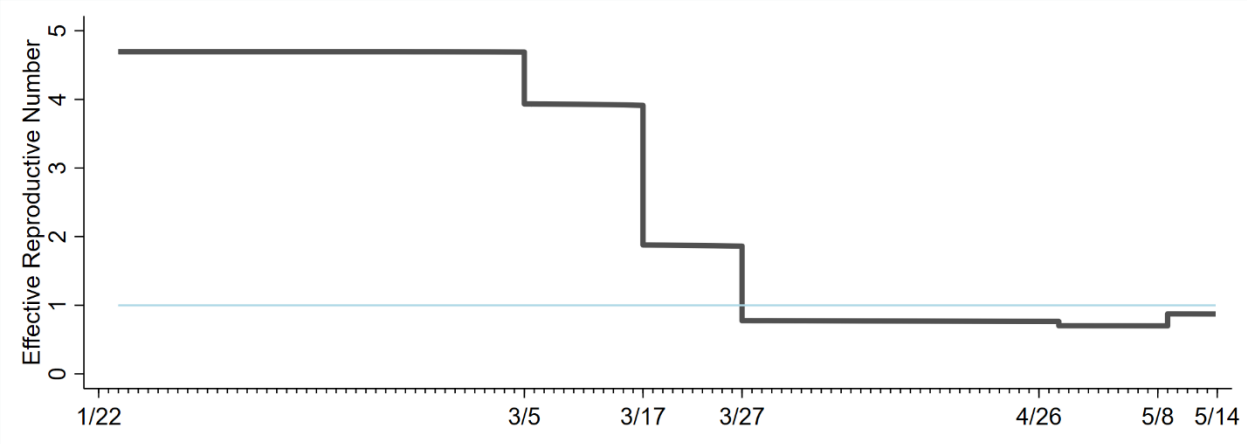

Figure 3. The estimated effective reproductive number in Colorado through May 14, describing the average number of infections directly generated by each case, plotted as a function of time. Blue line shows a reproductive number of 1. When the reproductive number is above 1, infections are increasing; when below 1, infections are decreasing.

Part 2. Estimated number of SARS-CoV-2 cases in Colorado to date and case detection rate.

We used the daily number of new symptomatic infections estimated by the SEIR model and compared that to the daily number of new cases reported by CDPHE. We estimate that the proportion of symptomatic cases reported to CDPHE in the past two weeks is 52% (Figure 4). We estimate that approximately 171,000 people in Colorado have been infected with SARS-CoV-2 through 5/15, or approximately 2.9% of the Colorado population. These estimates are based on our model projections through 5/15 using the parameter estimates in Table A1. Population-based sero-prevalence and/or infection surveys would increase confidence in these estimates and modeling assumptions.

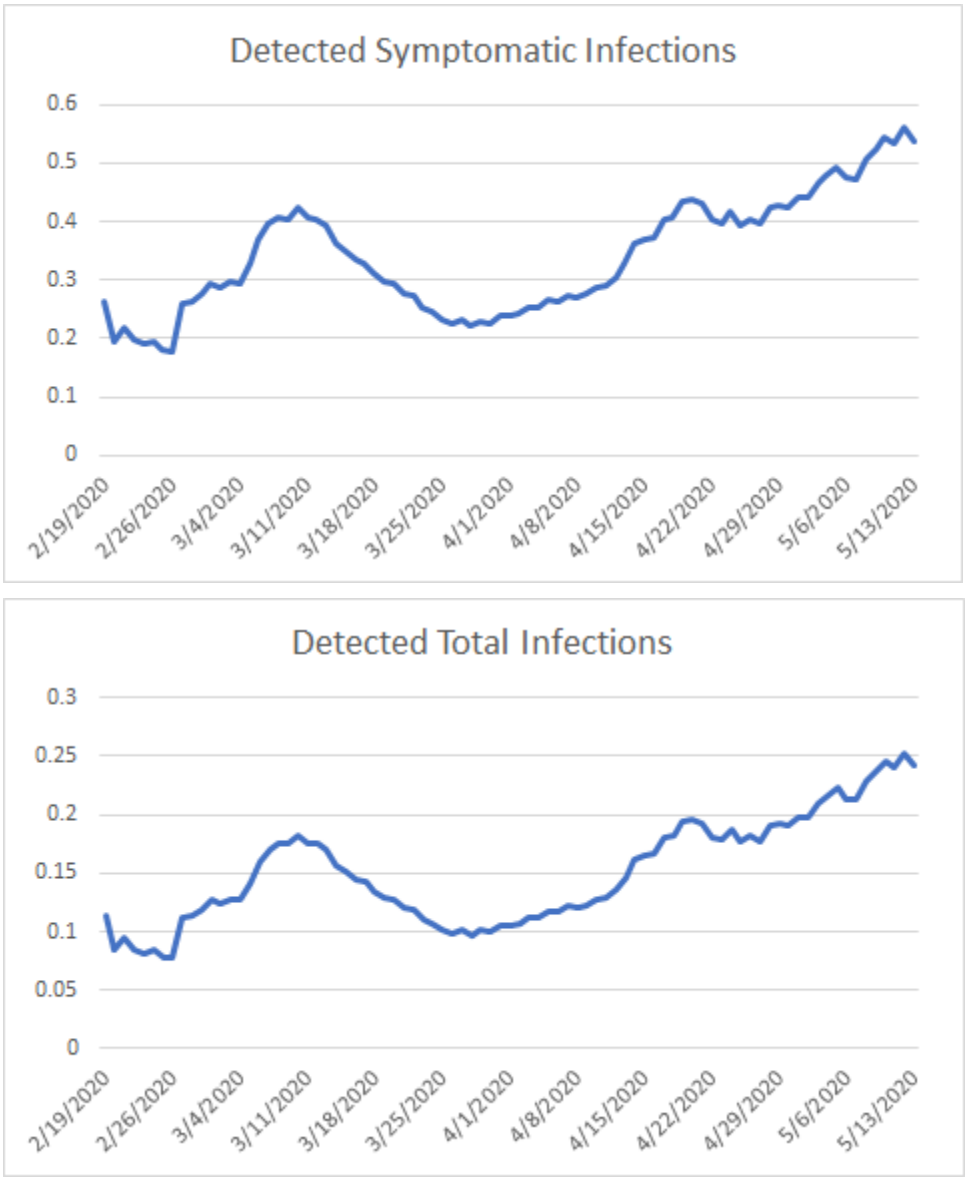

Figure 4. Estimated proportion of symptomatic COVID-19 cases (top) and all COVID-19 infections (including both asymptomatic and symptomatic infections) (bottom) that are being captured by Colorado state surveillance systems, over time. The estimated daily number of new cases is based on SEIR model outputs. The number of cases captured by state surveillance systems is the number of cases reported by CDPHE, using the onset date of symptoms or, if this is not available, the date of report minus 7 days based on typical lags.

Part 3. What do we project for the coming months?

We used the age-structured SEIR model to project the expected number of total hospitalizations and need for critical care in the coming months under a set of scenarios that include relaxation of social distancing, changes in mask wearing and recommendations that older populations maintain high levels of social distancing. The estimates are based on the most recent parameter fits from Part 1 and Methods Appendix Table A1 and the scenario assumptions described below. Additionally, we assume a gradual increase in case isolation as result of increased testing capacity. We assume that individuals who test positive will be more likely to self-isolate and, as a result of increases in testing going forward, the proportion of symptomatic individuals who choose to self-isolate will increase over time, rising by 5% weekly starting on June 1, with a maximum of 80% of symptomatic individuals self-isolating. Additionally, we assume social distancing is at 65% from May 15 to May 26 to account for potential reductions in social distancing corresponding with decreased public risk perception and gradual re-opening of different businesses, and following the trend from Phase 3 to Phase 4.

The first three sets of scenarios evaluate changes in social distancing and recommendations that older adults maintain high levels of social distancing. The fourth set of scenarios estimates the potential impact of increases in mask wearing in public.

Scenario 1. Population-wide decreases in social distancing starting on 5/27. These scenarios assume social distancing is reduced to 65% (A), 55% (B), 45% (C) indefinitely, or reduced to 55% on 5/27 and further reduced to 45% on 6/27 (D). Mask wearing is maintained at current levels and the relaxation of social distancing occurs across all age strata.

Scenario 2. Decreases in social distancing, with adults age 60+ maintaining high levels of social distancing. Scenario 2 uses the same reductions in social distancing (A-D) as scenario 1, with the additional assumption that adults age 60 and over maintain 80% social distancing indefinitely.

Scenario 3. Decreases in social distancing, with about half of adults age 60+ maintaining high levels of social distancing. Scenario 3 uses the same reductions in social distancing (A-D) as scenario 1, assuming approximately half of adults age 60 and over maintain 80% social distancing indefinitely.

Scenario 4. Increases in mask wearing in public. Scenario 4 considers the potential impacts of changes in mask wearing from 50% (Scenario 4A), to 65% (Scenario 4B), to 80% (Scenario 4C). These scenarios assume social distancing is reduced to 55% on 5/27 and that approximately half of adults age 60+ maintain high levels of social distancing. The intent is to illustrate the gain from mask wearing in one social distancing scenario.

These projections do not account for the potential impacts of contact tracing (discussed below) or any seasonal impacts on transmission.

Projected impact of changes in social distancing

As in our prior report, we project that hospital capacity will not be exceeded in the coming months if social distancing is held at 65%, mask wearing is maintained and case detection and isolation are gradually increased (Figure 5 and Table 1). We project hospital capacity will be exceeded this summer if social distancing is relaxed to 45%, mask wearing is maintained and case detection and isolation is gradually increased, even if older populations maintain high levels of social distancing. If social distancing is relaxed further to approximately 55%, older people will need to maintain high levels of social distancing in order to prevent exceedance of hospital capacity. in all scenarios, the epidemic curve

is rising at the vertical line at August 15, the time that schools, colleges, and universities begin to open. In most scenarios, we project the next peak in infections will be greater than the prior peak in April.

Older adults, age 60+ have accounted for approximately half of hospitalized COVID-19 cases in Colorado since late March although they comprise 21% of the Colorado population. Therefore reducing their risk of becoming infected with COVID-19 can reduce mortality overall and hospital demand (Figure 6). Notably, middle age adults also comprise a large proportion of hospitalized COVID-19 patients.

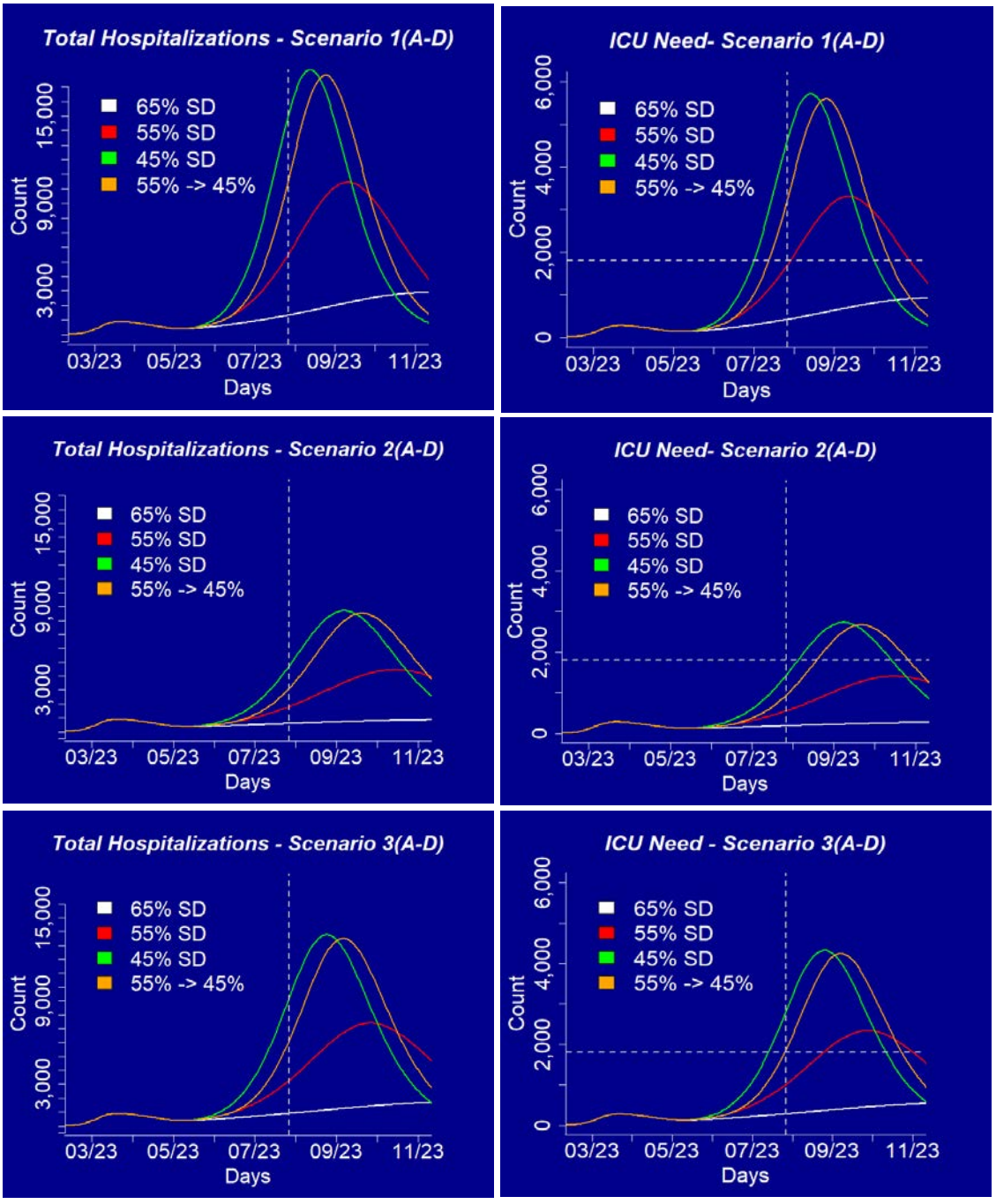

Figure 5. Projected total hospitalizations (left) and ICU needs (right) using the age-structured SEIR model for four scenarios that include relaxation of social distancing on 5/27 to 65%, 55%, 45% as well as a scenario that considers

relaxation of social distancing to 55% on 5/27 and to 45% on 6/27. These scenarios are modelled as population-wide changes in social distancing (top), changes in social distancing assuming older populations (age 60+) maintain high levels (80%) of social distancing (middle), and changes in social distancing assuming half of older populations (age 6+0) maintain high levels (80%) of social distancing (bottom). These projections assume that the indicated levels of social distancing are maintained indefinitely, mask-wearing is maintained at current levels, and case isolation gradually increases up to 80% of symptomatic cases self-isolating within 48 hours of infectiousness. Grey vertical dashed line indicates August 15, the approximate start of school. Horizontal dashed line indicates ICU threshold of 1,800 beds. All scenarios assume social distancing is at 65% from 5/15 to 5/26.

Table 1. Projected date that ICU bed capacity is reached in Colorado under model projections assuming different levels of social distancing starting May 27. ICU capacity is estimated at 1,800 critical care beds, a figure provided by CDPHE.

	Set 1 Population-wide decreases in social distancing	Set 2: Decreases in social distancing with high levels of social distancing (80%) by older adults (age 60+)	Set 3: Decreases in social distancing with high levels of social distancing by half of older adults
Scenario A: 65% SD*	N/A	N/A	N/A
Scenario B: 55% SD	08/15	N/A*	09/10
Scenario C: 45% SD	07/18	08/21	07/30
Scenario D: 55% SD in June, 45% July forward	07/30	09/04	08/12

*Under these scenarios, ICU capacity is not expected to be exceeded. ICU capacity of is fixed at 1,800 critical care beds, which accounts for increased ICU need as social distancing is relaxed and elective surgeries are resumed. These projections assume mask wearing is maintained at current levels and case detection and isolation is increased 5% per week up to 80% of symptomatic cases isolating.

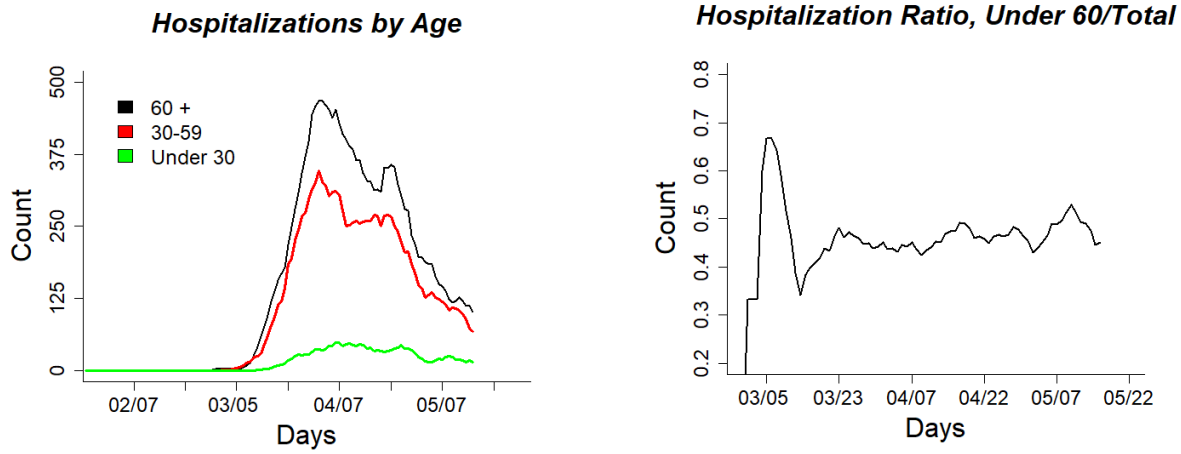

Figure 6. Daily number of hospitalizations for each age group (left) and ratio of under 60 hospitalizations to total hospitalizations (right) over time. Data are from CDPHE reported hospitalizations, using collection date as date of hospitalization.

Projected impact of changes in mask wearing

We project that increases in mask wearing in public have the potential to modestly decrease the number of hospitalizations and ICU demand at the peak, and delay the peak (Figure 7). Having data on the prevalence of mask wearing would be useful for modeling purposes, but more importantly for determining the need for efforts to increase the percentage of people wearing a mask when indicated.

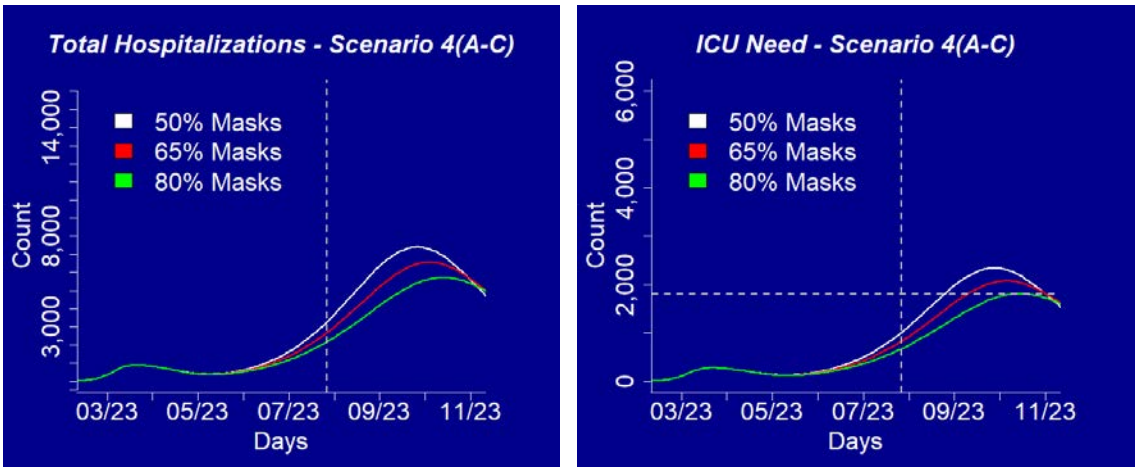

Figure 7. Projected total hospitalizations (left) and ICU needs (right) if mask wearing is held at 50% (white line), increased to 65% on 5/27 (red line), or 80% on 5/27 (green line). These projections assume 55% social distancing of individuals under 60 starting 5/27, half of individuals over 60 maintaining high (80%) social distancing, and case detection and isolation is increased 5% per week up to 80% of symptomatic cases isolating.

Implications for contact tracing

In order to assess the potential contact tracing needs in the near-term, we projected the number of incident cases of symptomatic COVID-19 per day under three different projections involving 65% social distancing (Figure 8). In all scenarios, we estimate there are approximately 500 new symptomatic cases per day currently and the number of incident cases is expected to grow in all scenarios. Given current contact tracing capacity estimates, this suggests we are already at capacity and will likely exceed capacity in the coming weeks.

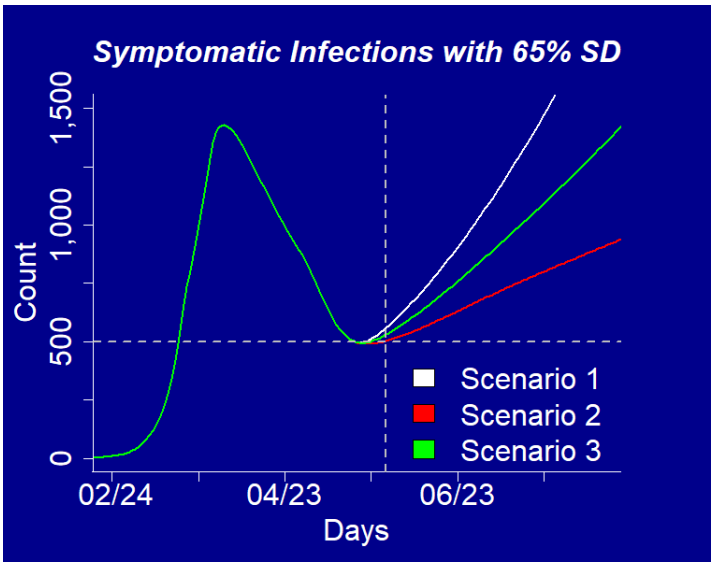

Figure 8. The projected number of incident (new) symptomatic cases per day, with dotted grey horizontal line to indicate projected capacity of contact tracing (500 cases per day) at 65% social distancing by all individuals (Scenario 1A, white line), 65% social distancing and adults age 60+ maintain high levels of social distancing (Scenario 2A, red line), and 65% social distancing and half of adults age 60+ maintain high levels of social distancing (Scenario 3A, green line).

Methods Appendix

In this report, we used the previously described age-structured SEIR model using Colorado-specific estimates of the probability of hospitalization and critical care need [1, 3, 4].

Model fitting and parameter estimation. We use established methods to fit reported COVID-19 hospitalizations to our model outputs in order to estimate parameters described in Table 1 as described in [1]. We used the CDPHE case reports to infer hospitalizations through 3/31 and the EMResource hospital census of hospitalizations starting 4/1.

Table A1. Model parameters estimated using our age-structured SEIR model, by fitting our model outputs to Colorado COVID-19 hospital census data, provided by CDPHE.

	Range of possible values	Fitted value	Fit using data through:
Social distancing*			
Phase 1. Estimated social distancing from mid-March until the start of the stay at home order, modeled as 3/17 - 3/25	10 – 70%	52%	05/11
Phase 2. Estimated social distancing during the state-wide stay at home order, modeled as 3/26 to 4/26	50 – 99%	80%	05/19
Phase 3. Estimated social distancing during the transition to state-wide Safer at Home, modeled as 4/27 to 05/08	50 – 99%	85%	05/27
Phase 4. Estimated social distancing during state-wide Safer at Home, modeled as 05/09 to 05/14	50-99%	75%	05/27
Mask wearing			
Proportion of individuals wearing masks, 4/4 to 4/26	0.1 - 0.5	0.40	05/11
Proportion of individuals wearing masks, 4/4 to 4/26		0.50	Assumed†
Case isolation			
Decrease in infectious - symptomatic contact rate due to self-isolation by symptomatic after March 5 (dividing by 0.57 gives proportion that self-isolate) ¶	0.3 - 0.8 (Ferguson et al)	0.30	05/11
Transmission parameters			
The rate of infection (beta)	0.2 - 0.6 (MIDAS**)	0.48	05/11
Ratio of infectiousness for symptomatic vs. asymptomatic individuals (lambda)	1.0 - 4.0 (Li et al., Zou et al.)	1.65	05/11

*Fit using This accounts for measures to encourage people stay at least 6 feet from people outside of their household as well as measures to promote handwashing and sanitizing to reduce fomite-based transmission.
†Given the temporal correlation of potential increases in mask wearing and changes in social distancing challenging parameter estimation, we assumed a 25% increase in mask wearing starting on 4/27.

¶Self-isolation by symptomatic cases is assumed to occur 48 hours after the onset of infectiousness and decrease the 76% of contacts that typically occur outside of the home [5], leading to a 57% decrease in contacts among those that self-isolate. This parameter jointly accounts for the percent of symptomatic individuals that self-isolate and the imperfect decline in contacts. Dividing the value in the table by 0.57 gives the proportion of symptomatics that self-isolate.
**The range of potential parameter estimate values were obtained from the MIDAS Online COVID-19 compilation of parameter estimates available here.

Estimating the effective reproductive number. We consider the subset of our SEIR model differential equations describing all compartments with infectious individuals (i.e., only Exposed, Infectious, and Asymptomatic). The Jacobian matrices describing only infection (F) and flux between compartments (V) were computed using Colorado-specific fitted parameters. The basic reproductive number R0 for a system is described by the spectral radius of the next generation matrix F*V^(-1) and the effective R0 as a function of the time course of the epidemic is depicted in Figure 3 [6]. Note that the discrete jumps correspond to state policy changes concerning social distancing.

Estimating the proportion of cases detected. In order to estimate the number of cases being detected by state surveillance systems, we compared model outputs of the daily number of symptomatic cases and all cases (symptomatic + asymptomatic) to the reported number of provided by CDPHE. We used this information to estimate proportion of symptomatic cases being detected each day. Reported case data were evaluated using the date of symptom onset. For cases with missing onset date, we imputed onset date by taking the date of report and subtracting 7 days based on typical lags. This comparison can be used to estimate the proportion of symptomatic cases detected by the state surveillance system over time, as well as the total number of infected individuals detected.

***Code*.** Code for our models is posted on Github: https://github.com/agb85/covid-19

Detailed description of scenarios modeled. The scenarios modeled are described in below.

Scenario	Social distancing by individuals <60, starting 5/27*	Social distancing by older adults (age>60), 5/15-5/26	Social distancing by older adults (age>60), 5/27-	Percent of population wearing masks
Scenario 1 Population-wide reductions in social distancing				
1A	65%	65%	65%	50%
1B	55%	65%	55%	50%
1C	45%	65%	45%	50%
1D	55% until 6/26, reduced to 45% 6/27	65%	55% until 6/26, reduced to 45% 6/27	50%
Scenario 2. Population-wide reductions in social distancing, older adults maintain high social distancing				
2A	65%	80%	80%	50%
2B	55%	80%	80%	50%
2C	45%	80%	80%	50%
2D	55% until 6/26, reduced to 45% 6/27	80%	80%	50%
Scenario 3. Population-wide social distancing, half of older adults maintain high social distancing				

3A	65%	72.5%	72.5%	50%
3B	55%	72.5%	Half at 80%, Half at 55% (modeled as SD=67.5%)	50%
3C	45%	72.5%	Half at 80%, Half at 45% (modeled as SD=62.5%)	50%
3D	55% until 6/26, reduced to 45% 6/27	72.5%	*5/27 to 6/27:* Half at 80%, Half at 55% (modeled as SD=67.5%) *6/27 forward:* Half at 80%, Half at 45% (modeled as SD=62.5%)	50%
Scenario 4. Mask wearing				
4A**	55%	72.5%	Half at 80%, Half at 55% (modeled as SD=67.5%)	50%
4B	55%	72.5%	Half at 80%, Half at 55% (modeled as SD=67.5%)	65%
4C	55%	72.5%	Half at 80%, Half at 55% (modeled as SD=67.5%)	80%

*Modeled with a start date of 5/27. From 5/15 to 5/26, social distancing is set at 65%.

**This is equivalent to scenario 3B.

All scenarios assume case detection and isolation is increased 5% per week starting June 1 to a max of 80% of symptomatic cases isolated.

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